Fisheye lens designs and their relative performance

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ABSTRACT

New panoramic and immersive digital imaging developments have generated increased interest in high performance fisheye camera lenses suitable for 35 mm single lens reflex (SLR) cameras.

Special concerns for such applications are the uniformity of illumination and radial image mapping. Because two hemispherical images are digitally stitched together to form a complete 360-degree x 180-degree image, the performance of the lens at the 90 degree (preferably more than 90 degree) edge of the fisheye image is just as important as the center of the image. Lateral color, high order distortion (edge compression) and severe drop-off of illumination at the full field become obvious image defects and cause seams in the immersive image.

Fisheye lens designs have widely varying relative illumination and distortion across the hemispherical field of view of the lens. After describing the contributing factors to relative illumination, we survey a collection of fisheye designs and compare their illumination performance, radial mapping and lateral color. A new method of measuring relative illumination and radial mapping in the laboratory is described and results on commercially available fish-eye lenses are presented.

Keywords: Fisheye lenses, optical design, relative illumination, distortion testing.

1. DEFINITION OF FISHEYE LENSES

1.1 Origin of the term "Fish-eye" and all sky lenses

Robert W. Wood originally coined the term "fish-eye" in his book Physical Optics[1] in the context of a discussion about the refraction of light and the refraction of rays entering the level surface of a pond. Wood goes on to describe a water filled pinhole camera (his "fish-eye" camera) that is capable of simulating the "fish-eye" view of the world. His homemade camera was functional and produced interesting examples of images that "embrace" 180 degree fields of view.

In "Photographic Lenses"[2] published in 1932, Willy Merté credits the first real "fish-eye" lens to R. Hill[3]. According to Kingslake[4], the Hill Sky Lens of 1924 was manufactured by Beck of London. Hill evidently is the first to develop a glass "fish-eye" lens, and his design was found to be useful to meteorologists for all-sky photography.

1.2 Definition of the fisheye optical design form

A fisheye lens generally has a front lens group of a far greater negative refractive power than that of an ordinary inverted telephoto type wide angle lens since a fairly large back focal distance, relative to the focal length of the whole lens system, is required for avoiding an increase in the size of the lens system. This extreme power distribution is apt to cause field curvature and astigmatism in the transmitted image. In addition, sagittal flare is increased if a high speed such as F/2.8 is desired. To improve the field curvature and astigmatism, it is necessary to avoid any negative deviation of the Peztval sum. This has been generally attempted by providing, at the image side of the aperture stop, at least one doublet composed of a positive lens element of a low-refractive-index-low-dispersion glass and a negative lens element of a high-refractive-index-high-dispersion glass with a cemented intermediate surface of a suitable negative refractive power formed there between. This approach helps avoid significant negative deviation of the Petzval sum while permitting correction of chromatic aberration.

1.3 Relative Illumination in Fisheye lenses

From Ray[5], the $\cos^4\theta$ law of image illumination states that at image point E, by inverse square law E $\cos^2\theta$, by Lambert's cosine law E $E_0 \cos\theta$; circular exit pupil becomes elliptical with its area reduced by factor $\cos\theta$, therefore E $E_0 \cos^4\theta$.

The power of the fisheye lens in wide angle, panoramic and immersive imaging is that the uncorrected barrel distortion is used to distribute the light flux over increasingly smaller areas towards the edges of the field of view. In this way, the fisheye lens is able to violate the $\cos^4\theta$ illumination law that affects the images of rectilinear lenses.

1.4 Fisheye lenses suitable for 35 mm photography

Fisheye lenses for 35 mm photography are typically found in two classes:

a. *Full frame fisheye lenses* capture a hemispherical image across the diagonal of the 35 mm frame. These lenses range in focal length from 14-16 mm and have a ratio of back focal distance to effective focal length of 2.2 to 2.4.

b. *Circular image (hemispherical) fisheye lenses* capture a full 180 degrees within the narrow height of the 35 mm film frame. The lenses in this class have effective focal lengths ranging from 6 to 8 mm and have a ratio of back focal distance to effective focal length of 4.55- 4.75.

2. 35 mm FISHEYE LENS DESIGNS

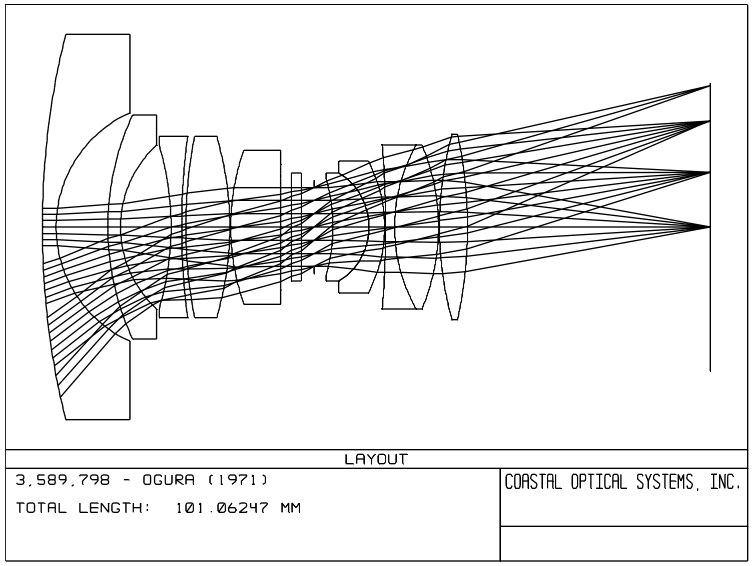
Fisheye lens design prescriptions suitable for 35 mm photography are not commonly presented in lens design texts or databases. For example, out of the 15,100 US patent designs, the leading optical design database LensVIEW™ (Optical Data Solutions, NY) does not include any 35 mm fisheye camera lenses US patents. The two Japanese patent examples listed below are from LensVIEW™. The designs presented in this paper have been collected from patent literature and entered into ZEMAX®(Focus Software). For the purposes of this review:

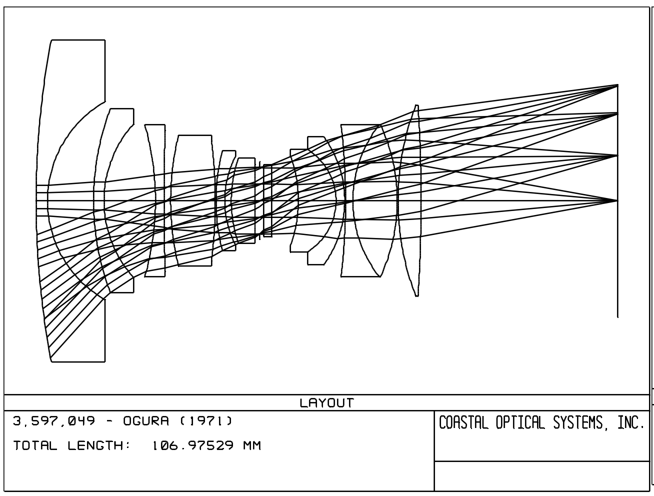
a. Only designs capable of greater than or equal to 180-degree field of view are considered.
b. Only designs capable of f / 2.8 operation are considered.
c. The designs have been scaled to their natural effective focal length that would be used for 35 mm photography (most patent prescriptions are presented with EFL=1 or 100).
d. Only the first embodiment is presented in the table.

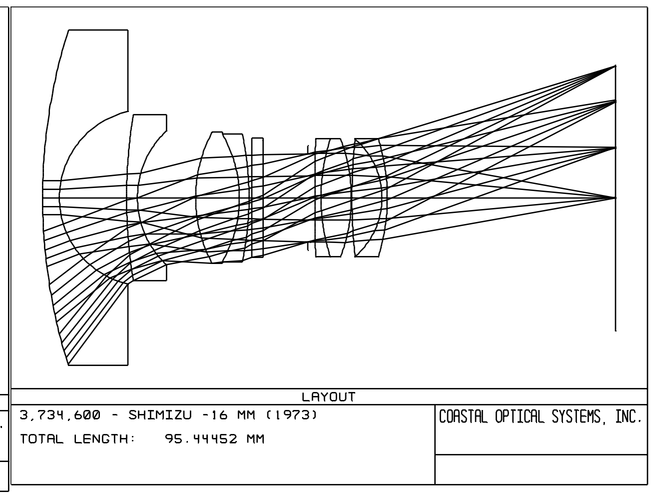
In most cases, information about lens apertures and vignetting is missing from the patent literature. In all cases, the location of the aperture stop is omitted. Each of these designs has been evaluated carefully by ray tracing, and we have made every attempt to carefully model the lens intended by the inventor. We have also made every effort to correct typographical errors (intentional or not) in the patents.

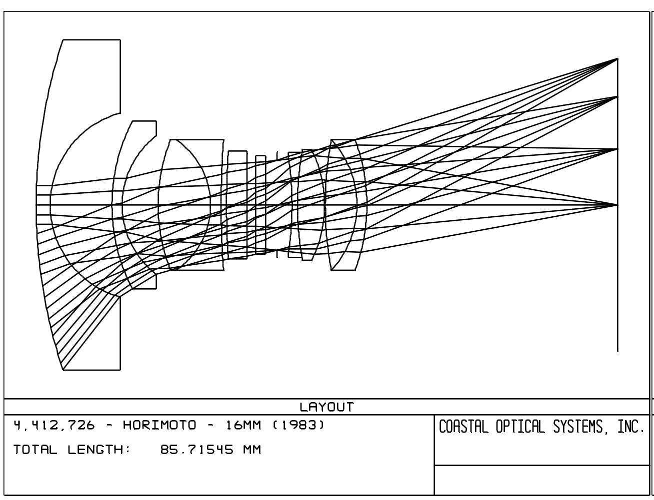
2.1 Full Frame Fisheyes

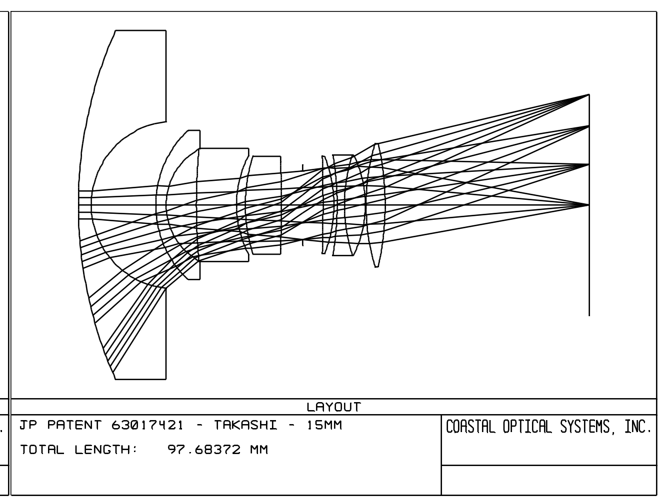
Patent Number (glass)	Inventor	Assignee	Patent Date	# of elements / # of groups	EFL	Total Length	Maximum Diameter
US 3,589,798	T. Ogura	Minolta	29 June 1971	10 / 8	16 mm	101.6	58.2
US 3,597,049	T. Ogura	Minolta	Aug. 3, 1971	11 / 9	16	107	58
US 3,734,600	Y. Shimizu	Nippon Kogaku	22 May 1973	8 / 5	16	95.5	59.5
US 4,412,726	M. Horimoto	Minolta	1 Nov. 1983	9 / 6	16	85.7	50.4
JP63-017421	Takashi	Canon	Jan 25, 1988	8 / 7	15	97.7	66.7











Full Frame Fisheye lenses (15-16 mm EFL)

2.2 Full Frame Fisheye performance

2.2.1 Full Frame Fisheye Relative Illumination

Real ray trace analysis was used to evaluate the relative illumination performance of the full frame fisheye lenses. The relative illumination calculations are discussed in detail in Rimmer[7]. All relative illumination calculations are performed at 587.6 nm. In many of these designs, vignetting is used by the optical designer to control off-axis aberrations because a half stop or full stop drop in relative illumination is tolerable in conventional photography.

Patent	Inventor	Relative Illumination @ 90º field of view
4,412,726	Horimoto	55.1%
3,589,798	Ogura	59.7%
63-017,421	Takashi	59.4%
3,597,049	Ogura	59.2%
3,734,600	Shimizu	52.1%

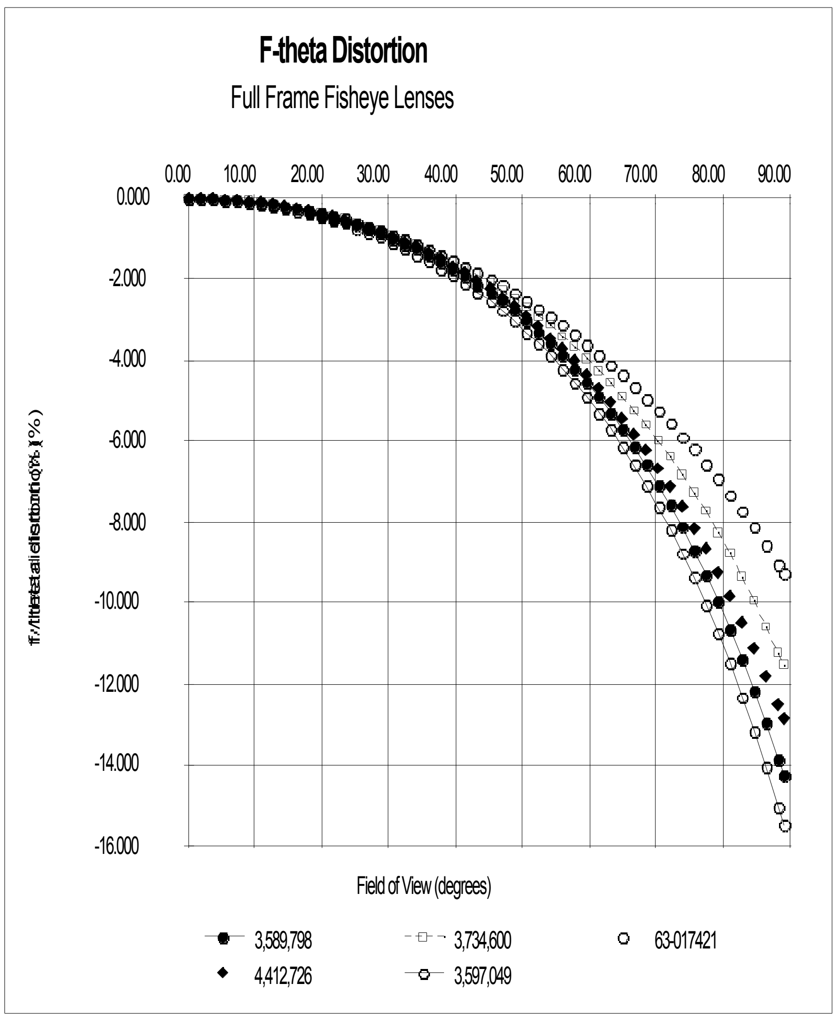
2.2.2 Distortion (departure from f-theta mapping)

The reference height of an undistorted ray in a lens that obeys F-Theta mapping is given by

$Y_{ref} = f$ _

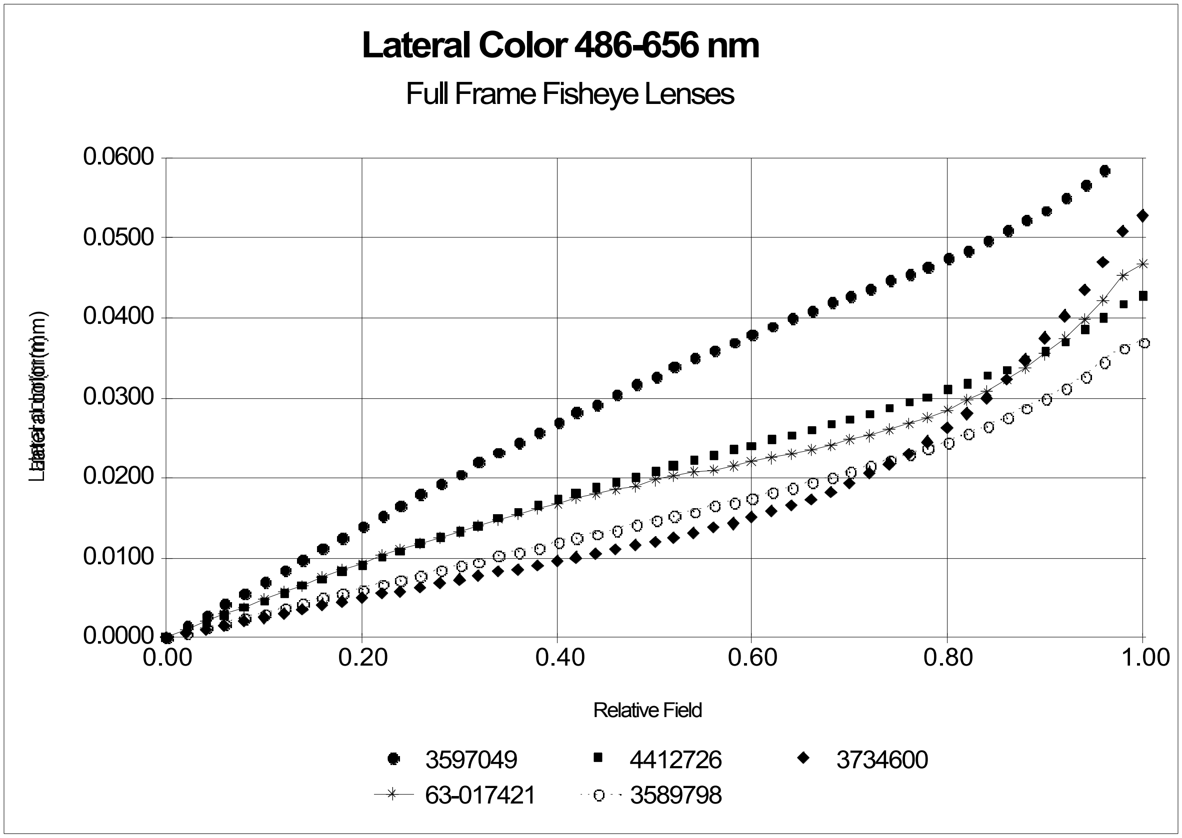
Where f is the focal length and _ is the angle in object space.

The adjacent plot shows departure from perfect linear (f-theta) mapping for each of the full field fisheye lenses.



2.2.3 Lateral color

Lateral color is the lateral shift on the image plane intersection between the shortest wavelength chief ray and the longest wavelength chief ray. The lateral is calculated by real ray trace analysis of the designs and is plotted as the difference between 486 nm and 656 nm.

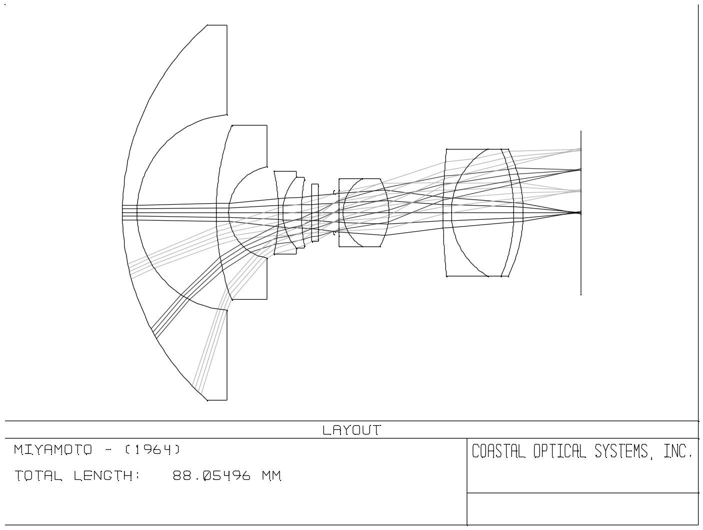


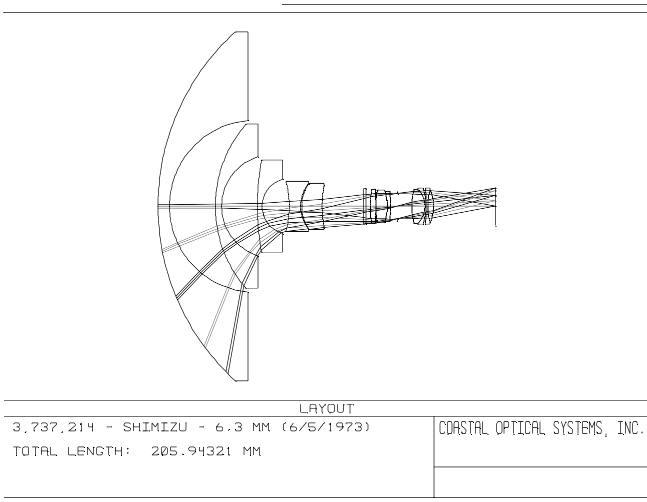
2.3 Circular Image Fisheyes

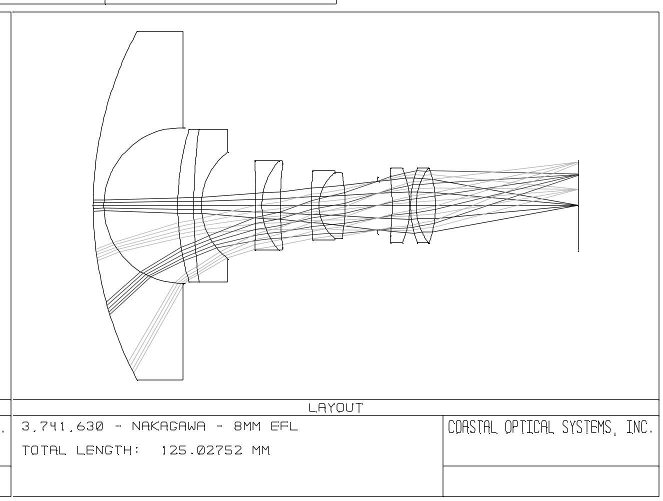
Patent Number	Inventor	Assignee	Date	# of elements / # of groups	EFL	Total Length	Maximum Diameter
n / a	Miyamoto	Nippon Kogaku	1964	9 / 5	8 mm	88	72 mm
US 3,737,214	Y. Shimizu	Nippon Kogaku	29 Sept. 1971	12 / 9	6.3	205.9	213
US 3,741,630	J. Nakagawa	Olympus Optical	26 June 1973	10 / 6	8	125	90
JP60-153018	Satoru	Asahi Optical	Aug. 12, 1985	10 / 7	8	139.7	81.6
n / a	Colucci	Coastal Optical	1997	11/7	7.45	174	150

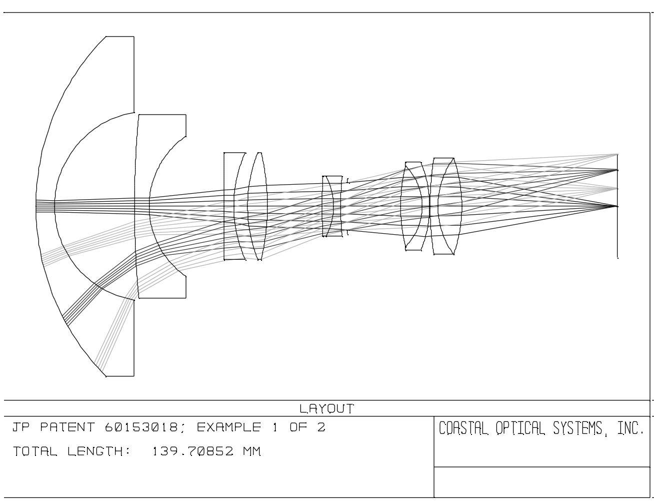
Note: total length = front vertex to image plane

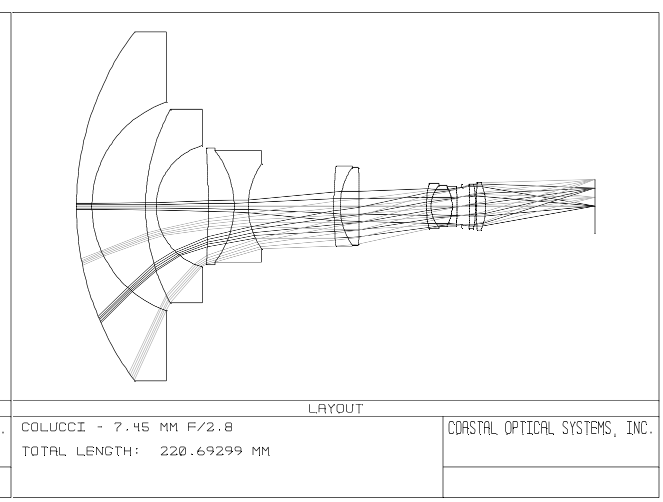
This list is not intended to be exhaustive – the lens designs presented are each unique and noteworthy for one or more reasons. Horimoto has at least two additional fisheye patents (4,009,943 and 4,256,373), but 4,412,726 has the best performance and operates at f/2.8. The 6 mm fisheye design by Shimizu is remarkable not only because of its 220-degree field of view, but because the three largest elements in the forward lens group in all three examples in the patent are made of BK7!











Circular Image Fisheye Examples

2.3 Circular Image Fisheye Performance

Patent Number	Inventor	# of elements # of groups	EFL	Full Field Relative Illumination	Full Field F-theta Distortion	Full Field Lateral Color
N /a	Miyamoto (Nikon)	9 / 5	8 mm	84.6%	-3.84%	0.016 mm
US 3,737,214	Y. Shimizu (Nikon)	12 / 9	6.3	87.7%	-3.14%	0.028
US 3,741,630	J. Nakagawa (Olympus)	10 / 6	8	92.7%	-9.75%	0.029
JP60-153018	Satoru (Asahi Optical)	10 / 7	8	70.9%	+0.02%	0.007
n / a	Colucci (Coastal Optical)	11/7	7.45	93.9%	-1.85%	0.017

3 MEASUREMENT OF RELATIVE ILLUMINATION AND RADIAL MAPPING

Fisheye photographic lenses are used for measurement of the angular position of points on the object hemisphere in areas of study such as woodland canopy studies, metrology, full-sky astrophotography and aurora surveys. In these areas, calibration of the relationship between the radial position of an image point and the object space angle is critical.

We have used a method similar to Herbert[8] to evaluate fisheye lenses that uses a high performance digital camera and digital image analysis techniques to evaluate the performance of the fisheye lens.

3.1 Test Apparatus

A circular table (Fig. 1) of 1.05-meter diameter was used for the radial image mapping tests. Mounted on the circumference of the table was a printed checker board pattern with alternating black and white squares, each measuring about 18-mm by 18-mm. These were designed to be 2° across. The center of the table was marked with a cross for positioning of the camera lens with its front nodal point at the center of the table. The use of the round table reduces the computational complexity of calculating fields-of-view from image data. Ideally, a larger round table would be preferred to reduce the effect of positioning and measurement errors.

The checkerboard pattern was generated in a PhotoShop (Adobe Systems, Inc., San Jose, CA) and scaled to the circumference of the table. The accuracy of the pattern is about 1% in local regions, and about 1° around the whole circumference. Because we assume radial symmetry of the optic, only a one-dimensional measurement was made.

The table was set in the middle of an interior room (no confounding outside light sources), slightly over 3-meters square. No special attempts have been made in the lighting, which was by two sets of three "warm white" 34-watt fluorescent bulbs. The fluorescent bulbs are used as a qualitative measure of symmetric ghosts and veiling glare in the lenses.

With the exception of the Nikkor (Nikon, Tokyo, Japan) 6mm, f/2.8 lens, the images were all acquired using the Nikon D1 digital camera. This camera was compatible with all but one of the tested lenses, freeing us from the vagaries of film development and image digitization. The digital camera offered high consistency of image capture, and an accurate digitally controlled shutter. This latter feature allowed us to generate a corrected gamma curve for the conversion of pixel values to relative illumination.

The Nikkor 6mm lens interfered with the body of the Nikon D1. We were thus relegated to shooting on film (Kodak Ektapress PJ100), scanned using a Polaroid Corporation (Cambridge, Mass.) SprintScan 35.

Because the uniformity of illumination could not be controlled, the data were gathered by rotating the camera around the nominal "nodal point" of the fisheye lens. Light fall-off values are all reported relative to that of the center of the fisheye lens image. The illumination of the neutral gray wall was found to be more uniform than that of the checkerboard tape, so that was used for calculation of illumination values.

PhotoShop (Adobe Systems, Inc.) was used to gather the data on both illumination uniformity, and for data on the radial mapping of the fisheye image. The value of the green channel was used throughout all these tests, though the conversion of all images to grayscale may be more rigorous.

Fisheye Measurement Laboratory, Nikkor 6mm f-2.8
Figure 1

Because of the discrete digital nature of the image acquisition, the accuracy of measurement for the width of a single block was poor, at about +/- 15%. Measurements were made over several blocks and averaged.

3.2 Image Acquisition Protocol

Multiple images were acquired with a reference lens with shutter speeds ranging from 0.5-seconds (1/2s) to 0.004-seconds (1/250s). From these images a gamma curve was reconstructed for both the D1 and for the film/scanner combination. This was used to correct the illumination fall-off data for all the lenses shot with that camera. Additional images were acquired, at several different f-stops, with the lens pointed at different angles relative to the reference line of the test room. The data presented are generally shot with an aperture full open, and 2 stops down from full open (f/5.6 for the f/2.8 lenses and f/8 for the f/4 lenses). (Note: the Coastal Optical 4.88mm lens is a fixed aperture lens.)

4 TEST RESULTS OF CIRCULAR IMAGE FISHEYE LENSES

Seven commercially available circular image fisheye lenses were tested in this study. Each of these lenses produce a complete circular image on 35 mm film format covering at least 180 degrees field of view. All of the lenses tested are compatible with the f-mount on the Nikon D1 camera. In all cases only a single lens of a given type was tested (past experience with a number of units of one of the lenses in this list showed large unit-to-unit variations).

Circular Image Fisheye Lenses tested

Manufacturer	Focal Length	Aperture	Field of view	# of elements # of groups	Weight	Dia. x Length
Coastal Optical	7.45 mm	f/2.8	185˚	11 elements	3.63 kg	171 x 174
Coastal Optical	4.88 mm	f/5.2	185˚	10 / 6	0.50	83 x 101
Nikon	8 mm	f/2.8	180˚	10 / 8	1.00	123 x 128
Nikon	6 mm	f/2.8	220˚	12 / 9	5.20	236 x 160
Peleng	8 mm	f/3.5	~180˚	(?)	0.37	73.5 x 65
Sigma (-'98)	8 mm	f/4.0	180˚	12 / 8	0.48	73.5 x 59.5
Sigma ('99-)	8 mm	f/4.0	180˚	10 / 6	0.32	73.5 x 61.8

4.1 Relative Illumination

All percent illuminations are relative to illumination at the center of the field of view, corrected for room illumination uniformity.

Lens		Field Angle 80°	88°	90°	100°
Coastal Optical	7.45mm @ f/2.8	85%	82%	81%	
Coastal Optical	7.45mm @ f/5.6	88	85	84	
Coastal Optical	4.88mm @ f/5.2	95	91	90	
Nikon MF	8 mm @ f/2.8	80	73	64	
Nikon MF	8 mm @ f/5.6	97	95	74	
Nikon MF	6 mm @ f/5.6	95	94	94	85
Peleng	8 mm @ f/3.5	74	63	59 @ 89°	
Peleng	8 mm @ f/5.6	89	78	69 @ 89°	
Sigma (-’98)	8 mm @ f/4.0	53	42	39	
Sigma (-’98)	8 mm @ f/8.0	78	68	63	
Sigma EX(‘99-)	8 mm @ f/4.0	67	52	49	
Sigma EX(‘99-)	8 mm @ f/8.0	72	62	54	

Note the improvement in uniformity at higher f-numbers, as one would expect. For those lenses with large changes between 88° and 90° there are also large lateral color shifts apparent in these regions. The Peleng 8mm fisheye lens creates a circle that is slightly larger than 24 mm diameter. The film gates in modern 35mm cameras crops the top and bottom of the image circle. In addition there is a circular chuck mark on the inside hemispherical surface of the front element, with an equivalent diameter of 172° causing image degradation and scattered and stray light problems.

4.2 Radial Image Mapping

The radial image mapping for these lenses can be fit to the general form

$$r = \alpha * \sin(\beta \theta)$$

where r is the distance from the center of the camera image to the point of interest, θ is the angle between the central axis of the fisheye lens and the line to the point of interest in the real image, α is a scale factor, in these cases to convert from angle in space to millimeters in the image plane, and β is the radial mapping parameter. (It should be noted that β effects α very strongly.) The theoretical fisheye map of $r = \alpha\theta$ is approached as β approaches zero.

The results are presented with all lenses focused at infinity and the aperture set 2 stops above fully open. Though the aperture does not affect image size it does affect field of view. In some of these lenses, the field of regard changes with the aperture setting because the part of the lens barrel that is limiting the field of view changes location when the iris is adjusted. The focus does affect image size slightly, the image size growing as the lens focus is reduced from infinity.

The compression at 90 degrees is calculated as a simple ratio of the radial size of the checkerboard square at the edge of the field over the size of the checkerboard square at the center of the field.

$$\text{Image compression @ 90 degrees} = \frac{\text{Size of target at 90 degrees field}}{\text{Size of target at center of field}}$$

Radial Image Mapping

Lens	**Focal Length**	**Aperture**	**Compression @ 90 degrees**	**_**	**_**
Coastal Optical	7.45mm	f/5.6	108%	14.8*_	
Coastal Optical	4.88 mm	f/5.2	83%	11.4	0.46
Nikon MF	8 mm	f/5.6	83%	17.3	0.46
Nikon MF	6 mm	f/5.6	102%	12.5*_	
Peleng	8 mm	f/3.5	92%	23.8	0.34
Sigma (-'98)	8 mm	f/8.0	71%	14.9	0.54
Sigma EX ('99-)	8 mm	f/8.0	70.5%	14.7	0.54

The Coastal Optical 7.45 mm lens and the Nikon 6 mm exhibit almost perfect linear (*f* _) mapping. The 2% and 8% enlargement (magnification) of the targets at the edge of the field may be a result of the movement of the pupil towards the front of the lens as you move to the edge of the field (see Laikin[9]). In these two large lenses, this displacement of the pupil position is significant relative to the total object distance, introducing a change in the "effective" field angle of the checkerboard target. Every attempt was made to orient the large fisheye lenses with the best approximation of the nodal plane in the center of the table and on the axis of rotation.

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