Testing Fabry-Perot etalons

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The monochromatic filters we use for observing the solar chromosphere are built from Fabry-Perot etalons, consisting of two plane, parallel, highly reflecting surfaces separated by a small gap. These surfaces must be of excellent optical quality and it comes as no surprise that the filters are expensive gear. The more advanced "professional" or "research grade" models are extremely expensive and we may wonder whether an extra $7000 for choosing a 0.3 Å filter instead of a 0.5 Å is worth the money. How can we make sure of the actual performance of our filters ?

We explain here an elementary testing method for Fabry-Perot etalons which can be implemented with a modest equipment and some care. When illuminated by a broad source of monochromatic light the etalon produces a system of interference rings. Measuring them will allow estimating the two main features of the etalon, its full width at half maximum (FWHM) and its free spectral range (FSR). Their ratio is the finesse of the etalon.

1 The transmission factor of a Fabry-Perot etalon

Light entering the etalon undergoes mutiple-beam interference, due to the great number of reflexions on the etalon surfaces. As a result most wavelengths are strongly attenuated in the outgoing light while some others are much better transmitted, in spite of the high reflecting factor of the etalon surfaces. Because of this property etalons can be used as monochromatic filters.

We call **transmission factor** of the etalon the ratio of the transmitted intensity divided by its maximum possible value. This factor T is given by the following formula (the "Airy function"), explained in optics textbooks :

$$T = \frac{1}{1 + F \sin^2\left(\frac{2\pi nh}{\lambda}\cos r\right)}, \qquad (1)$$

with the following notation (see figure 1) :

- $F = 4R/(1-R)^2$, where R is the reflectivity of the etalon surfaces
- n is the refractive index of the gap ($n = 1$ for an air-spaced etalon, $n \approx 1.6$ for a mica-spaced etalon)
- h is the gap spacing, expressed with the same unit as the light wavelength λ, whence $(2\pi nh\cos r)/\lambda$ in radians
- r is the angle of refraction inside the etalon, related to the angle of incidence i by Snell's law $\sin i = n \sin r$ (or simply[1] $r \approx i/n$ if i is small).

[1] We shall always write $a = b$ for an exact formula and $a \approx b$ for an approximate equality.

For a given etalon F, n and h are fixed. Only λ, i (and r) may vary ; the corresponding variation of T will be discussed below. The **maximum transmission factor** $T = 100\%$ is obtained when $\sin^2(...) = 0$, that is

$$T = 100\% \text{ when } \frac{2nh}{\lambda}\cos r = k \tag{2}$$

where $k = 1, 2, 3, ...$ is a positive integer. In practice k will be in the order of a few hundreds. The **half maximum** $T = 50\%$ is obtained when $F\sin^2(...) = 1$, that is $\sin(...) = \pm 1/\sqrt{F}$

$$T = 50\% \text{ when } \frac{2nh}{\lambda}\cos r = k \pm \varphi, \text{ with } \varphi = \frac{1}{\pi}\arcsin\frac{1}{\sqrt{F}}. \tag{3}$$

In practice the reflectivity R is 90% at least, thus F is large ($F > 360$) and we may take the approximate value $\varphi \approx 1/\pi\sqrt{F} = 1/(2\mathcal{F})$, where $\mathcal{F}$ is an important feature of the etalon, called its **finesse** and defined by

$$\mathcal{F} = \frac{\pi}{2}\sqrt{F} = \frac{\pi\sqrt{R}}{1-R}. \tag{4}$$

2 The etalon as a monochromatic filter

When the etalon is used as a monochromatic filter, the light from a given point of the Sun hits it under some angle of incidence i and we want to know which wavelengths of this light are best transmitted by the filter. For the sake of simplicity we consider here normal incidence, thus $i = 0$, $r = 0$ and $\cos r = 1$; to deal with other values of i it would suffice to replace the thickness h by $h\cos r$. We are interested in the variations of the transmission factor as a function $T(\lambda)$ of λ only, as given by (1).

The graph of $T(\lambda)$ is the familiar comb-shaped curve (figure 2). Its teeth correspond to the wavelengths of maximum transmission. Combining the etalon with a **blocking filter** will select a single tooth of the comb and constitute a true monochromatic filter. The maxima $T = 100\%$ are obtained for specific wavelengths λ_k given by (2) and the wavelengths $\lambda_k^{\pm}$ of 50% transmission are given by (3), namely

$$\lambda_k = \frac{2nh}{k}\ ,\ \lambda_k^- = \frac{2nh}{k+\varphi}\ ,\ \lambda_k^+ = \frac{2nh}{k-\varphi} \tag{5}$$

where k is a positive integer.

The interval between the consecutive teeth numbered k and $k+1$ is called the **free spectral range** (FSR) ; the larger it is, the easier it will be to isolate a single wavelength with a blocking filter. From (5) we obtain

$$FSR = \lambda_k - \lambda_{k+1} = \frac{\lambda_k\lambda_{k+1}}{2nh} \approx \frac{\lambda_k^2}{2nh}. \tag{6}$$

In practice λ_{k+1} is close to λ_k and we can use the latter approximate value.

The width of a single tooth is called the **fullwidth at half maximum** (FWHM) ; the smaller it is, the narrower will be the transmitted band... and the higher the price. From (5) we have

$$FWHM = \lambda_k^+ - \lambda_k^- = \frac{4nh\varphi}{k^2-\varphi^2} \approx \frac{\lambda_k^2}{2nh\mathcal{F}}. \tag{7}$$

In practice k and F are large, φ^2 is negligible compared to k^2 and we can use the approximate value. An important relation then follows from (6) and (7) :

$$\mathcal{F} \approx \frac{FSR}{FWHM}\ , \tag{8}$$

ratio of the etalon's FSR to its FWHM, which explains the name "finesse" given to the number $\mathcal{F}$.

3 The etalon on a test bench

Here the etalon is not illuminated by the Sun but by a broad monochromatic source, a hydrogen spectral lamp of wavelength $\lambda = 6562.8$ Å if we are testing a Hα filter. As above, each ray hitting the etalon with an angle of incidence i (such as the ray 0 in figure 1) undergoes multiple reflections inside it and gives rise to multiple outgoing parallel rays 1, 2, 3,... in the same direction i. Interference of these rays creates a dark or bright spot in this direction, which can be imaged with a camera lens focused at infinity, placed behind the etalon (figure 3). The camera sensor will show a pattern of concentric bright rings (figure 4), each ring being produced by all rays from a direction of maximal transmitted intensity.

Thus we are now interested in the variations of the transmission factor as a function $T(i)$ of i only, as given by (1) for a fixed λ (the wavelength of the spectral lamp). Scanning a profile of the picture along a diameter will show the graph of $T(i)$, still comb-shaped (figures 5 and 6) though different from the previous one. Careful measurements of this profile allow to compute the etalon's FSR and FWHM as we now explain.

The filter is often tuned on a wavelength slightly greater than Hα so that tilting it by some (small) angle i_1 will tune it on Hα. The graph of $T(i)$ then looks like figure 5 ; the first maximum is obtained for $i = i_1$, which gives the radius of the first ring of the pattern. If our filter is exactly tuned on the Hα line the wavelength λ of the lamp coincides with one of the λ_k's of §2. Then $i_1 = 0$ and the graph of $T(i)$ looks like figure 6.

Let $d_1, d_2, \dots$ be the diameters of the successive bright rings on an image taken with a lens of focal length f. Considering the first rings only, the corresponding angles of incidence $i_1, i_2, \dots$ are small and we may use approximate formulas as follows :

– measure the diameters $d_1, d_2, \dots$
– compute the angles $i_m \approx d_m/2f$ for $m = 1, 2, \dots$(in radians)
– compute $\Delta_m = i_{m+1}^2 - i_m^2$
– the free spectral range of the etalon is then $FSR \approx \lambda\Delta_m/2n^2$.

Let us explain this procedure. The refracted angles are $r_m \approx i_m/n$ and, according to (2), the number

$$\frac{2nh}{\lambda}\cos r_m \approx \frac{2nh}{\lambda}\left(1 - \frac{i_m^2}{2n^2}\right)$$

must be an integer if the incidence i_m produces a bright ring. The next bright ring will correspond to the next integer and, taking the difference,

$$\Delta_m = i_{m+1}^2 - i_m^2 \approx \frac{n\lambda}{h}. \tag{9}$$

This number Δ_m may be seen as an evaluation of the distance between the consecutive rings numbered m and $m+1$, in analogy with the free spectral range of §2. In our approximation (valid for the first few rings) it has the same value for $m = 1, 2, ...$; we may therefore replace a single Δ_m by the average of $\Delta_1, \Delta_2, ...$ to reduce the measurement errors. It is related to the **free spectral range** by the approximate formula

$$FSR \approx \frac{\lambda}{2n^2} \Delta_m \ , \tag{10}$$

which follows from (6) and (9), the λ_k's of §2 being close to the wavelength λ of the lamp.

A similar method gives the FWHM. We need here the diameters d_m^+ and d_m^- of the first rings, measured at half maximum height on the ring profile (figure 5) :

- measure the diameters $d_1^-, d_1^+, d_2^-, d_2^+, ...$
- compute the angles $i_m^+ \approx d_m^+/2f$ and $i_m^- \approx d_m^-/2f$ for $m = 1, 2, ...$ (in radians)
- compute $\delta_m = (i_m^+)^2 - (i_m^-)^2$
- the FWHM of the etalon is then $FWHM \approx \lambda\delta_m/2n^2$.

Let us explain. According to (3), the number

$$\frac{2nh}{\lambda} \cos r_m^{\pm} \approx \frac{2nh}{\lambda} \left(1 - \frac{(i_m^{\pm})^2}{2n^2} \right)$$

must be an integer plus or minus $\varphi \approx 1/(2\mathcal{F})$ where $\mathcal{F}$ is the finesse. Then, taking the difference between the $+$ and $-$ expressions,

$$\delta_m = \left(i_m^+\right)^2 - \left(i_m^-\right)^2 \approx \frac{n\lambda}{h\mathcal{F}}. \tag{11}$$

This number δ_m may be seen as an evaluation of the thickness of the m-th ring, in analogy with the FWHM of §2. In our approximation (valid for the first few rings) it has the same value for $m = 1, 2, ...$; we may therefore replace a single δ_m by the average of $\delta_1, \delta_2, ...$ to reduce the measurement errors. It is related to the **full width at half maximum** by the approximate formula

$$FWHM \approx \frac{\lambda}{2n^2} \delta_m \ , \tag{12}$$

which follows from (7) and (11). Comparing the above equations we check that

$$FSR/FWHM \approx \Delta_m/\delta_m \approx \mathcal{F}$$

as obtained in (8).

Conclusion. Knowing the index n of the gap ($n = 1$ for an air-spaced etalon, $n \approx 1.6$ for a mica-spaced etalon) and the wavelength λ of the spectral lamp, measuring the diameters d_m of bright rings gives its FSR by (10) and its spacing h by (9). Measuring the diameters $d_m^{\pm}$ at half maximum gives its FWHM by (12) and its finesse by (11).